Quadratic Regression

**Remember to Turn on your scatter plot by going to [2nd] Y=**

Enter the information by going to [STAT] [Enter] __________ __________ __________

Find the Regression Equation by going to [STAT] ⇒ 5:QuadReg, (comma)
vars/Y-Vars/Function/Y1

View your scatter plot by going to [ZOOM] 9: ZoomStat

To evaluate, use the table. (Input x-value)

1. Individual's Retirement Fund

   The following table gives the average amount, in thousands of dollars, of an individual's retirement fund.

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>9</td>
</tr>
<tr>
<td>1990</td>
<td>19</td>
</tr>
<tr>
<td>1995</td>
<td>45</td>
</tr>
<tr>
<td>2000</td>
<td>101</td>
</tr>
<tr>
<td>2005</td>
<td>196</td>
</tr>
</tbody>
</table>

(a) Use this information to construct a quadratic regression to represent the model rounding all constants to 3 decimal places.

   Use $x = 1$ for 1985, $x = 2$ for 1986, ....

   $y = 0.571x^2 - 3.451x + 14.251$

(b) To the nearest thousand dollars, what will the fund be worth in 2010?

   $\$311,000$

2. Sales of TV Antennas

   The total sales, $S$, of TV antennas for various years from 1980 to 1995 are shown in the table below, where $t = 0$ represents the year 1980. Sales are shown in millions of dollars.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>76.3</td>
</tr>
<tr>
<td>5</td>
<td>82.2</td>
</tr>
<tr>
<td>7</td>
<td>84.6</td>
</tr>
<tr>
<td>13</td>
<td>80.9</td>
</tr>
<tr>
<td>15</td>
<td>77.3</td>
</tr>
</tbody>
</table>

(a) Determine the quadratic regression equation that models this data.

   [Round coefficients to the nearest thousandth.] $y = -193x^2 + 3.341x + 76.921$

(b) Using the regression equation found, determine in what year sales reached their maximum.

   $8.65 \rightarrow 1988$

(c) Use the regression equation to estimate the total sales of TV antennas for 2008. [Round the answer to the nearest tenth of a million.]

   $2008 - 1980 = 28 \times 3$

   $12.6 \text{ million}$
3. Average Cost of new Sedan
The following table gives the average cost, to the nearest hundred, of a new 4-door sedan.

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>$12,800</td>
</tr>
<tr>
<td>1994</td>
<td>$15,500</td>
</tr>
<tr>
<td>1997</td>
<td>$19,200</td>
</tr>
<tr>
<td>2000</td>
<td>$24,300</td>
</tr>
<tr>
<td>2003</td>
<td>$30,100</td>
</tr>
</tbody>
</table>

(a) Use this information to construct a quadratic regression to represent the model, rounding all constants to 3 decimal places.
Use $x = 1$ for 1991, $x = 2$ for 1992, ...

$$y = 60.317x^2 + 602.222x + 1223.175$$

(b) Using this regression model, estimate during which year the average cost of a new 4-door sedan reached $37,000.

$$37,000 = 60.317x^2 + 602.222x + 1223.175 - 37000$$

4. Sales of new T-Shirt
Sales of a new T-shirt style are shown in the table below. These sales were recorded at two-month intervals for one year and the values for sales, $S$, of the new T-shirt style are given in thousands of dollars.

(a) Write a regression equation with coefficients rounded to the nearest hundredth.

$$y = -.88x^2 + 8.57x + 47.7$$

(b) Using this regression equation, estimate, to the nearest thousand dollars, sales for month 11 of this year. $S = 31,000$

4. Quadratics
5. Write the equation for each graph in intercept form for the information given.

a. \[ y = a(x - h)^2 + k \]

b. x-intercepts: $(-2, 0), (2, 0)$
   Point: $(-4, 0)$

   \[ y = a(x + 2)(x - 2) \]
   \[ 8 = a(-4 + 2)(-4 - 2) \]
   \[ 8 = a(-2)(-6) \]
   \[ 8 = -12a \]
   \[ a = \frac{8}{12} = \frac{2}{3} \]

   \[ y = \frac{2}{3}(x + 2)(x - 2) \]

   \[ y = a(x + 2)(x - 2) \]
   \[ -4 = a(2 + 2)(2 - 2) \]
   \[ -4 = a(4)(-4) \]
   \[ a = \frac{4}{16} = \frac{1}{4} \]

   \[ y = \frac{1}{4}(x + 2)(x - 2) \]

   \[ y = a(x + 3)(x - 3) \]
   \[ 18 = a(-3 + 3)(3 - 3) \]
   \[ 18 = a(2)(-3) \]
   \[ a = -2 \]

   \[ y = -2x(x + 3) \]

   \[ 18 = a(-3 + 3)(x - 3) \]
   \[ 18 = a(2)(x - 3) \]
   \[ 18 = -6a \]
   \[ a = -3 \]
6. Write the equation for each graph in vertex form for the information given.

a. Vertex(-2, -4): Point: (1, 2)
   \[ y = a(x + 2)^2 - 4 \]
   \[ 2 = a(1+2)^2 - 4 \]
   \[ a = a(9) - 4 \]
   \[ 6 = 9a \]
   \[ \frac{6}{9} = a \]
   \[ a = \frac{2}{3} \]
   \[ y = \frac{2}{3}(x + 2)^2 - 4 \]
   \[ (1, 3) \]

b. \[ y = a(x - 1)^2 + 3 \]
   \[ 5 = a(0 - 1)^2 + 3 \]
   \[ 5 = a(1) + 3 \]
   \[ 2 = a \]
   \[ y = 2(x - 1)^2 + 3 \]

7. Write the quadratic function in standard form.

a. \((-1, -3),(1, 5),(2, 3)\)
   \[ y = -2x^2 + 4x + 3 \]

b. \((-2, -1),(1, 1),(2, 27)\)
   \[ y = 3x^2 + 7x + 1 \]

c. \((-3, -4),(-1, 0),(9, -10)\)
   \[ y = -\frac{1}{4}x^2 + x + \frac{5}{4} \]

d. \((-6, 46),(2, 14),(4, 56)\)
   \[ y = \frac{5}{2}x^2 + 6x - 8 \]
Carefully graph each of the following. Then, evaluate the graph at any specified domain value.

8. \( f(x) = \begin{cases} 
  x + 5 & \text{if } x < -2 \\
  x^2 + 2x + 3 & \text{if } x \geq -2 
\end{cases} \)

- \( f(3) = 3^2 + 2(3) + 3 = 18 \)
- \( f(-4) = -4^2 + 2(-4) + 3 = 1 \)
- \( f(-2) = (-2)^2 + 2(-2) + 3 = 3 \)

9. \( f(x) = \begin{cases} 
  2x + 1 & \text{if } x \geq 1 \\
  x^2 + 3 & \text{if } x < 1 
\end{cases} \)

- \( f(-2) = (-2)^2 + 3 = 7 \)
- \( f(6) = 2(6) + 1 = 13 \)
- \( f(1) = 2(1) + 1 = 3 \)

10. \( f(x) = \begin{cases} 
  x^2 - 1 & \text{if } x \leq 0 \\
  2x - 1 & \text{if } 0 < x \leq 5 \\
  3 & \text{if } x > 5 
\end{cases} \)

- \( f(-2) = (-2)^2 - 1 = 3 \)
- \( f(0) = 0^2 - 1 = -1 \)
- \( f(5) = 2(5) - 1 = 9 \)
11. \[ f(x) = \begin{cases} x^2 & x \leq 0 \\ -x^2 + 4 & x > 0 \end{cases} \]

\[ f(-4) = (-4)^2 = 16 \]
\[ f(0) = 0^2 = 0 \]
\[ f(3) = -(3)^2 + 4 = -9 + 4 = -5 \]

12. Write equations for the piecewise functions whose graphs are shown below.

a. \[ f(x) = \begin{cases} (x + 3)^2 & -3 \leq x \leq -1 \\ \frac{-3}{2}x + \frac{5}{2} & -1 < x \leq 3 \end{cases} \]

b. \[ f(x) = \begin{cases} \frac{1}{3}x + 2 & 1 \leq x \leq 3 \\ (x-4)^2 + 2 & 3 < x \leq 6 \\ -x + 12 & x > 6 \end{cases} \]