Chapter 1 Notes: Quadratic Functions

*(Textbook Lessons 1.1 – 1.2) Graphing*

<table>
<thead>
<tr>
<th>Quadratic Function</th>
<th>A function defined by an equation of the form,</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>

The graph is a U-shape called a_______________.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>axis of symmetry:</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>y-intercept:</td>
</tr>
<tr>
<td>Vertex Form</td>
<td>• vertex is</td>
</tr>
<tr>
<td></td>
<td>• axis of symmetry is</td>
</tr>
<tr>
<td>Intercept Form</td>
<td>• x-intercepts are</td>
</tr>
<tr>
<td></td>
<td>• axis of symmetry is between (p, 0) and (q, 0)</td>
</tr>
</tbody>
</table>

**Axis of Symmetry**

The axis of symmetry of a parabola is a symmetry that divides the parabola into symmetry always passes through symmetry of the parabola.

How does $a$ affect the graph of a quadratic function?

| Maximum or Minimum Value of a Quadratic Function | The graph of $f(x) = ax^2$ where $a \neq 0$, opens and has a minimum when $a > 0$. The graph opens and has a maximum when $a < 0$. |
|-------------------------------------------------|---------------------------------------------------------------------------------
|                                                  | The graph of $f(x) = ax^2$. |
|                                                  | If $|a| > 1$, the graph of $f(x) = x^2$ is |
|                                                  | If $0 < |a| < 1$, the graph of $f(x) = x^2$ is |
Example 1:

\[ y = \frac{1}{2}x^2 - x - 6 \]

Form: \[ a = \_ \quad b = \_ \quad c = \_ \]

Find Axis of Symmetry:

Find the Vertex

Maximum or Minimum value?

y-intercept

What is known from \( a \)?

Example 2:

\[ y = -(x + 5)^2 + 2 \]

Form: \[ \_ \_ \_ \_ \_ \_ \_ \]

Vertex: \[ \_ \_ \_ \_ \_ \_ \_ \]

Axis of Symmetry: \[ \_ \_ \_ \_ \_ \_ \_ \]

What is known from \( a \)?

Maximum or Minimum value?

Change into Standard Form:

Example 3:

\[ y = 2(x - 3)(x + 1) \]

Form: \[ \_ \_ \_ \_ \_ \_ \_ \]

x-Intercepts: \[ \_ \_ \_ \_ \_ \_ \_ \_ \]

Axis of Symmetry: \[ \_ \_ \_ \_ \_ \_ \_ \_ \]

Vertex: \[ \_ \_ \_ \_ \_ \_ \_ \_ \]

What is known from \( a \)?

Maximum or Minimum value?

Change into Standard Form: \[ y \text{-Intercept?} \_ \_ \_ \_ \_ \_ \_ \_ \]
**Word Problem Example 4:**
Suppose that a group of high school students conducted an experiment to determine the number of hours of study that leads to the highest score on a comprehensive year-end exam.

The exam score \( y \) for each student who studied for \( x \) hours can be modeled by

\[
y = -0.853x^2 + 17.48x + 6.923
\]

Which amount of studying produced the highest score on the exam? What is the highest score the model predicts?

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**Finding Maximums and Minimums in the Graphing Calculator. (p.10)**

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**Textbook Lessons 1.3 – 1.4**

**Solve Equations by Factoring:** Factoring and the Zero Product Property can be used to solve many equations of the form \( x^2 + bx + c = 0 \).

**Intercept Form is Factored Form:** solutions are

The first step in **Factoring** is to factor out the GCF ( ) if the GCF is \( \neq 1 \! \)

**Example 1**) \( 16x^2 + 10x \)

\( 3x^2 - 6x - 4 + 2x \)

**Example 2**) \( x^2 - 7x + 12 \)

\( 2x^2 - 8x - 42 \)**
### Factoring a Trinomial of the Form $a x^2 + bx + c$ where $a \neq 1$

Example 3)

$$3x^2 - 5x - 2$$

<table>
<thead>
<tr>
<th>Difference of Squares</th>
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</table>

Example 4)

$$x^2 - 16 \quad 8x^2 - 18$$

<table>
<thead>
<tr>
<th>Perfect Square Trinomial</th>
</tr>
</thead>
</table>

Example 5)

$$x^2 + 24x + 144 \quad 8x^2 - 24x + 18$$

Because of the **Zero Product Property**, the factors of a quadratic help us find the **roots** of a quadratic function.

$$f(x) = x^2 + 2x - 3$$
Using a Graphing Calculator to solve quadratic equations having approximate solutions.

*Word Problem

A painter is making a rectangular canvas for her next painting. She wants the length of the canvas to be 4 ft. more than twice the width of the canvas. The area of the canvas must be 30 ft².

What should the dimensions of the canvas be?

A fraction that contains a radical in its denominator can be written as an equivalent fraction with a rational denominator (a denominator without a radical).

Never leave a radical in the denominator of a fraction. Always rationalize the denominator.

(Textbook Lessons 1.5) Solving Quadratic Equations by Finding Square Roots

Example 1: Review: Simplifying Radicals

\[ \sqrt{50} \quad \sqrt{24} \quad 4\sqrt{3} \cdot \sqrt{21} \]

Review: Rationalizing the Denominator

\[ \frac{\sqrt{7}}{2} \quad \frac{\sqrt{49}}{5} \]
When there is more than one term in the denominator, the process is a little tricky. You will need to multiply the numerator and denominator by the denominator's conjugate. The conjugate is the same expression as the denominator but with the opposite sign in the middle, separating the terms.

Example: Simplify \( \frac{2}{5 + \sqrt{3}} \)

Example: Simplify \( \frac{5}{4 - \sqrt{5}} \)

Example 2:
Solve: \( 3 - 5x^2 = -9 \)

Example 3:
Solve: \( 3(x - 2)^2 = 21 \)
The height \( h \) (in feet) of an object \( t \) seconds after it is dropped can be modeled by the function 
\[ h = -16t^2 + h_0 \]
where \( h_0 \) is the object’s initial height.

**Word Problem Example 4:** How long will it take an object dropped from a 550-foot tall tower to land on the roof of a 233-foot tall building?

\[ (Textbook Lessons 1.6) \quad Complex \ Numbers \]

The imaginary number: 
\[ i = \sqrt{-1} \]

\[ i^2 = _____ \]
\[ i^3 = _____ \]
\[ i^4 = _____ \] ….

What is \( i^{22} = _____ \)?

What is \( i^{31} = _____ \)?

What is \( i^{1321} = _____ \)?

Simplify \( \sqrt{-2} \) \quad \( \sqrt{-12} \) \quad \( \sqrt{-36} \)

Example 1: Solve \[ 2x^2 + 26 = -10 \] \quad \[ 0 = 4(x - 3)^2 + 1 \]
Complex Number: $a + bi$, where $b \neq 0$

- $a$ is the real number part
- $b$ is the imaginary number part

Example 2: **Plotting** Complex Numbers in the Complex Plane
Finding **Absolute Values** of Complex Numbers (Find the DISTANCE from the Complex # to the origin)

Plot $-3 + 2i$
Plot $3 - 5i$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find $|-2i|$
Find $|3 + 4i|$

Find $|-1 + 5i|$

Example 3: Adding and Subtracting Complex Numbers

$(-7 + 5i) + (12 + 3i)$  
$(2 + 3i) + (-8 - 6i)$  
$(-7 + 5i) - (12 + 3i)$  
$(4 + 10i) - (-12 + 20i)$

Example 4: Multiplying Complex Numbers

$-i(3 + i)$
$(2 + 3i)(-6 - 2i)$

Conjugates $(1 + 2i)(1 - 2i)$
Example 5: Dividing Complex Numbers

\[
\frac{-2}{3+5i} \quad \frac{6-7i}{3i} \quad \frac{2-7i}{1+i}
\]

Example 6: Equate Complex Numbers

Find the values of \(a\) and \(b\) that make the equation \(a + 4 + (2b - 6)i = 7 + 9i\) true.

\[\text{Conjugate Zeros Theorem}\] tells us that if \(a + bi\) is a zero of \(f(x)\), then \(a - bi\) is also a zero.

Find all zeros of \(f(x) = x^2 + 1\).

Example: Write a standard form quadratic equation that has the solution \(-3 + i\)


**ELECTRICITY** The impedance in one part of a series circuit is $1 + 3j$ ohms and the impedance in another part of the circuit is $7 - 5j$ ohms. Add these complex numbers to find the total impedance in the circuit.

**ELECTRICITY** Using the formula $E = IZ$, find the voltage $E$ in a circuit when the current $I$ is $3 - j$ amps and the impedance $Z$ is $3 + 2j$ ohms.

Find the total impedance for the following circuits.

\[
\begin{array}{c}
\text{Component and symbol} \\
\text{Resistor} & \text{Inductor} & \text{Capacitor} \\
\text{Resistance or reactance} & R & L & C \\
\text{Impedance} & R & Li & Ci \\
\end{array}
\]

---

**Textbook Lessons 1.7) Completing the Square** Creating Perfect Square Trinomials to Solve Quadratic Equations.

<table>
<thead>
<tr>
<th>Standard Form of a Perfect Square Trinomial:</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Factored Form of a Perfect Square Trinomial:</td>
<td></td>
</tr>
<tr>
<td>Rewrite as:</td>
<td></td>
</tr>
</tbody>
</table>

Examples: Solve $2x^2 - 36x + 162 = 32$ $x^2 + 10x + 25 = 108$
Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

$x^2 + 12x + c$

$x^2 - \frac{5}{3}x + c$

**STEPS:**
1. Put the equation in quadratic form $ax^2 + bx + c = 0$.
2. Divide every term by $a$. This will make $a = 1$.
3. Move $c$ to the other side of the equation.
4. Take "b", divide by 2, square it, and add to both sides of the equation.
5. Factor the "perfect square" trinomial on the left side as $(x + c)^2$ where $c = b/2$.
6. Simplify the right side.
7. Take the square root of both sides. Remember to put $\pm$ in front of the radical on the right side.
8. Simplify the radicals if necessary. Left side will be $x + c$ and right side will have the $\pm$ symbol.
9. Solve the equation for the variable.

**Equation with Imaginary Solutions**

Solve $4x^2 - 2x + 7 = 0$ by completing the square.
Example 1) Write: $y = x^2 - 8x + 11$ in vertex form. (Note: $a = 1$)

Example 2) Write: $y = 5x^2 - 20x - 24$ in vertex form. (Note: $a \neq 1$)

Changing from Standard Form $y = ax^2 + bx + c$ into Vertex Form by Completing the Square
Word Problems
STOPPING DISTANCE The formula \( d = 0.05s^2 + 1.1s \) estimates the minimum stopping distance \( d \) in feet for a car traveling \( s \) miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes?

(Textbook Lessons 1.8) The Quadratic Formula and the Discriminant
Let's take the standard form for a quadratic equation and solve for \( x \) (by completing the square):

\[
y = ax^2 + bx + c
\]
Quadratic Formula

The solutions of \( ax^2 + bx + c = 0 \), with \( a \neq 0 \), are given by

**Solve:**

\[
x^2 + 2x = 3 \\
4x^2 + 8x + 7 = 4 \\
6x^2 - 12x + 1 = 0
\]

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>The expression under the radical sign, called the.</th>
<th>in the Quadratic Formula is</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b^2 - 4ac &gt; 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b^2 - 4ac = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b^2 - 4ac &lt; 0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Describe the nature of the solutions of each quadratic before you solve by calculating the discriminant!

\[ 2x^2 = -6x - 7 \quad \quad \quad \quad \quad \quad \quad \quad x^2 - 6x + 9 = 0 \]

**Note:** Often with respect to polynomial functions, zeros are also called roots!

**Solutions/Zeros/Roots** of a polynomial may be real or unreal numbers. However, **x-intercepts are just real numbers** (where the polynomial crosses the x-axis on the Cartesian plane: a plane consisting of only real **ordered pairs**!)

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The height \( h \) (in feet) of an object \( t \) (in seconds) after it is launched or thrown can be modeled by the function \( h = -16t^2 + v_0t + h_0 \) where \( v_0 \) is initial velocity, \( h_0 \) is the object’s initial height.

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A man tosses a penny up into the air above a 100-feet deep well with a velocity of 5 \( ft/sec \). The penny leaves the man’s hand at a height of 4 ft. How long will it take the penny to reach the bottom of the well?
Ex. 2

A ball is hit straight up from a height of 4 ft. with an initial velocity of 60 ft/sec.

a. Write an algebraic representation

b. Find the number of seconds to reach the maximum height?

c. What is the maximum height of the ball?

d. How long does it take for the ball to hit the ground?

e. When will the height reach 40 feet?
Graph Quadratic Inequalities To graph a quadratic inequality in two variables, use the following steps:

1. Graph the related quadratic equation \( y = ax^2 + bx + c \).
   Use a line for < or >; use a line for \( \leq \) or \( \geq \).

2. Test a inside the parabola.
   If it satisfies the inequality, shade the region the parabola; otherwise, shade the region the parabola.

\[
y > x^2 - 8x + 17
\]

\[
y \geq x^2 - 4
\]
\[
y < -x^2
\]
Solve Quadratic Inequalities  Quadratic inequalities in one variable can be solved graphically or algebraically.

| Graphical Method | To solve $ax^2 + bx + c < 0$:  
First graph $y = ax^2 + bx + c$. The solution consists of the $x$-values for which the graph is below the $x$-axis.  
To solve $ax^2 + bx + c > 0$:  
First graph $y = ax^2 + bx + c$. The solution consists of the $x$-values for which the graph is above the $x$-axis. |

$0 \geq 2x^2 + 4x$

| Algebraic Method | Find the roots of the related quadratic equation by factoring, completing the square; or using the Quadratic Formula.  
2 roots divide the number line into 3 intervals.  
Test a value in each interval to see which intervals are solutions. |

Use a Sign Chart (Sign Analysis) to Solve. Express solution in Interval Notation.

- $x^2 + 2x < 0$  
- $-2x^2 + 5x \leq -12$  
- $(x - 4)^2(> 0)$

$x^2 - 4x + 7 \leq 0$
Extension: Revisit Modeling with Quadratic Functions

Example 1: Writing Quadratic Function in Vertex Form

Example 2: Writing a Quadratic Function in Intercept form

Example 3: Writing a Quadratic Function in Standard Form

This parabola passes through (−4,1), (0,9), and (1,16).

This parabola passes through (3,3), (6,12), and (9,27).
Example 4: Using **Quadratic Regression** to Find a Model

A study compared the speed \( x \) (in miles per hour) and the average fuel efficiency \( y \) (in miles per gallon) for cars. The results are shown in the table. Use your graphing calculator to find a quadratic model in standard form for the data.

<table>
<thead>
<tr>
<th>Speed, ( x )</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Efficiency, ( y )</td>
<td>22.3</td>
<td>25.5</td>
<td>27.5</td>
<td>29</td>
<td>28.8</td>
<td>30</td>
<td>29.9</td>
<td>30.2</td>
<td>30.4</td>
<td>28.8</td>
<td>27.4</td>
<td>25.3</td>
</tr>
</tbody>
</table>

Best-Fitting Quadratic Model is: 

Predict Fuel Efficiency for a speed of 75 mph. 

Find the speed that maximizes a car's fuel efficiency. 

**Revisit Piecewise Functions**

Graph \( f(x) = \begin{cases} x^2 - 3, & x < 0 \\ -x^2 + 2, & x \geq 0 \end{cases} \) 

Graph \( f(x) = \begin{cases} (x+4)^2 - 1, & -6 \leq x < -3 \\ -x^2 - 2x, & -3 \leq x < 4 \\ 5, & x \geq 4 \end{cases} \)