Unit 0: Extending Algebra 1 Concepts

What is a Function?

Definition:

---Function Notation---

Example: \( f(x) = x^2 - 1 \)

Interval Notation

A convenient and compact way to express a set of numbers on the real number line. We will use interval notation to represent the domain and range of a relation.

| When using interval notation, the symbol: | \( 2 \leq x < 6 \) as an inequality. |
| ( means "not included" or "open". | \([2, 6)\) in interval notation. |
| [ means "included" or "closed". |

For some intervals it is necessary to use combinations of interval notations to achieve the desired set of numbers. Consider expressing in interval notation, the set of numbers which contains all numbers less than 0 and also all numbers greater than 2 but less than or equal to 10.

| As an inequality: | \( x < 0 \) or \( 2 < x \leq 10 \) |
| In interval notation: | \((-\infty, 0) \cup (2, 10]\) |
Domain and Range

Domain:

___________________________________________________________________________
___________________________________________________________________________

Range:

___________________________________________________________________________
___________________________________________________________________________

Example: Domain and Range Word Problem

Tara’s car travels about 25 miles on one gallon of gas. She has between 10 and 12 gallons of gas in the tank.

a) List the independent and dependent quantities.

b) Find the reasonable domain and range values.

c) Write the reasonable domain and range in interval notation.

Continuity: A function is continuous over an interval of its domain if its hand-drawn graph over that interval can be sketched without lifting the pencil from the paper.

Function? Y N Continuous? Y N
Domain? ___________ Range? ___________

State the intervals in which the function is increasing, decreasing, or constant. Use Interval Notation.

Increasing: ______________________________
Decreasing: ______________________________
Constant: _______________________________

\( f(1) = \) _______ \( f(-1) = \) _______

For what number(s) \( x \) is \( f(x) = 1 \) ______________________
For what number(s) \( x \) is \( f(x) = 3 \) ______________________

What is the y-intercept? _______ What is the x-intercept? _______

For what numbers \( x \) is \( f(x) < 0 \)? ______________________
For what numbers is \( f(x) > 0 \)? ______________________
State the intervals in which the function is increasing, decreasing, or constant. Use Interval Notation.

Increasing: __________________________

Decreasing: __________________________

Constant: __________________________

\[ f(-4) = \quad f(3) = \]

For what number(s) \( x \) is \( f(x) = -1 \) __________________________

For what number(s) \( x \) is \( f(x) = 1 \) __________________________

What is the \( y \)-intercept? ______ What is the \( x \)-intercept? ______

For what numbers \( x \) is \( f(x) < 0 \) ? __________________________

For what numbers \( x \) is \( f(x) > 0 \) ? __________________________

**Example:** State whether the following relations are functions (continuous or not?). Give the domain and range of the following relations. State the intervals in which the function is increasing, decreasing, or constant. Use Interval Notation.

- **Function? Y N**
- **Continuous? Y N**
- **Domain:** ______
- **Range:** ______
- **Increasing:** ______
- **Decreasing:** ______
- **Constant:** ______

- **Function? Y N**
- **Continuous? Y N**
- **Domain:** ______
- **Range:** ______
- **Increasing:** ______
- **Decreasing:** ______
- **Constant:** ______

- **Function? Y N**
- **Continuous? Y N**
- **Domain:** ______
- **Range:** ______
- **Increasing:** ______
- **Decreasing:** ______
- **Constant:** ______

- **Function? Y N**
- **Continuous? Y N**
- **Domain:** ______
- **Range:** ______
- **Increasing:** ______
- **Decreasing:** ______
- **Constant:** ______
Piecewise Functions

A piecewise function is a function that is a combination of one or more functions. The rule for a piecewise function is different for different parts, or pieces, of the domain. For instance, movie ticket prices are often different for different age groups. So the function for movie ticket prices would assign a different value (ticket price) for each domain interval (age group).

EX. Graph each of the following. Identify whether or not he graph is a function. Then, evaluate the graph at any specified domain value.

1. \[ f(x) = \begin{cases} 
-2x + 1 & x \leq 2 \\
5x - 4 & x > 2 
\end{cases} \]

\[ f(-4) = \] 
\[ f(8) = \] 
\[ f(2) = \]
You have a summer job that pays time and a half for overtime. That is, if you work more than 40 hours per week, your hourly wage for the extra hours is 1.5 times your normal wage of $7.

a. Write a piecewise function that gives your weekly pay \( P \) in terms of the number of hours \( h \) you work.

b. How much will you get paid if you work 45 hours? _________________
Write Piecewise functions from a graph

Write equations for the piecewise functions whose graphs are shown below. Assume that the units are 1 for every tic marc.

3. \[ f(x) = \begin{cases} \text{__________} & \text{Domain: __________} \\ \text{__________} & \text{Range: __________} \end{cases} \]

4. \[ f(x) = \begin{cases} \text{__________} & \text{Domain: __________} \\ \text{__________} & \text{Range: __________} \end{cases} \]

5. \[ f(x) = \begin{cases} \text{__________} & \text{Domain: __________} \\ \text{__________} & \text{Range: __________} \end{cases} \]

Given the following piecewise function, graph and then write the domain and range in interval notation:

\[ f(x) = \begin{cases} -x & x < -1 \\ -2 & -1 \leq x < 2 \\ 2x & x \geq 2 \end{cases} \]

Domain: __________
Range: __________
Graphing and Solving Systems of Linear Inequalities

Process to Solving a System of Inequalities:

1) Graph each inequality shading one side of the line or other function.

2) The solution is the overlapping shaded regions of the graphs.

3) For < and > symbols, graph of line will be a dotted line.
   For ≤ and ≥ symbols, graph of line will be a solid line.

4) Choose a point to plug into each inequality. If the point makes a true statement, shade toward the point. If the point makes a false statement, shade away from the point.

Note: Any point in the shaded area will make a_____ statement in_______ of the original inequalities!!!
Linear Inequality Word Problems

**DRAMA:** The drama club is selling tickets to its play. An adult ticket costs $15 and a student ticket costs $11. The auditorium will seat 300 ticket-holders. The drama club wants to collect at least $3630 from ticket sales.

a. Write and graph a system of four inequalities that describe how many of each type of ticket the club must sell to meet its goal.

   ________________________________
   ________________________________
   ________________________________
   ________________________________

b. List three different combinations of tickets sold that satisfy the inequalities.

   ________________________________
   ________________________________
   ________________________________

**HEALTH:** Mr. Flowers is on a restricted diet that allows him to have between 1600 and 2000 Calories per day. His daily fat intake is restricted to between 45 and 55 grams. What daily Calorie and fat intakes are acceptable?
**Linear Programming**

**Linear Programming** is a technique that identifies the minimum and maximum value of some quantity, the quantity modeled by the *objective function*. Limits on the variables in the objective function are *constraints*, or restrictions (your inequalities).

Here are the Steps to Linear Programming:

1. **GRAPH the Constraints**

2. **Find Vertices of Feasible Region**

3. **Test Vertices of the Feasible Region in Objective Function** to find max and min.

---

**Step 1: GRAPH constraints**

**Example:**

**Objective Function:**

\[ P = 5x + 10y \]

**Constraints:**

\[ \frac{3}{2}x - 3 \geq y \]
\[ y < -x + 7 \]
\[ x \geq 0 \]
\[ y \geq 0 \]

The intersection of the shaded region is called the feasible region and it is the area that contains all the solutions to the system.

**Step 2: Find vertices of the feasible region**
Step 3: Test vertices of feasible region in the Objective Function

The Vertex Principle of Linear Programming states there is a maximum or minimum value of the linear objective function (ALWAYS a PROFIT OR COST Function) at one or more of the vertices of the feasible region.

Linear Programming Word Problems

Suppose you are selling cases of mixed nuts and roasted peanuts. You can order no more than a total of 500 cans and packages and spend no more than $600. Use the information in the table below to determine the following:

How can you maximize your profit?

How much is the maximum profit?

Define Variables:

<table>
<thead>
<tr>
<th>Mixed Nuts</th>
<th>Roasted Peanuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 cans per case</td>
<td>20 packages per case</td>
</tr>
<tr>
<td>YOU PAY.....$24 per case</td>
<td>YOU PAY.....$15 per case</td>
</tr>
<tr>
<td>SELL AT.....$3.50 per can</td>
<td>SELL AT.....$1.50 per pkg.</td>
</tr>
<tr>
<td>How much do you pay per can?</td>
<td>How much do you pay per pkg.?</td>
</tr>
<tr>
<td>_____________</td>
<td>_____________</td>
</tr>
<tr>
<td>Profit on each can =</td>
<td>Profit on each pkg. =</td>
</tr>
<tr>
<td>_____________</td>
<td>_____________</td>
</tr>
</tbody>
</table>

State Constraints:
Step 1: GRAPH the constraints

Step 2: Find vertices of the feasible region
(May need to find intersections, x-intercepts, y-intercepts mathematically if you can’t accurately see them on graph)

Step 3: Test vertices of feasible region in the Objective Function
Press 2nd CATALOG, holding down the down arrow until DiagnosticOn is highlighted. Press ENTER ENTER.

**Calculator Directions for Linear* Regression**

*(TI-83, TI-83 Plus, or TI-84 Plus)*

**BEFORE YOU BEGIN:**
- Clear out (or de-highlight) any equations in the Y= editor (Y₁, Y₂, Y₃, etc.)

**STEP 1: Entering in the data into two lists (L₁ and L₂)**
- Hit **STAT**
- Choose 1:Edit by either hitting **ENTRY** or **ENTER**.
  - If necessary, clear out any old data in the lists:
    - Use **Δ** to get cursor to cover L₁ at top of list; press CLEAR ENTER. Repeat process for L₂.
- Type the data values for the independent (x) variable in column L₁. Hit **ENTRY** after each entry.
- When you finished entering data in L₁, hit **2ND** and then enter the data values for the dependent (y) variable in column L₂.

**STEP 2: Making the scatterplot**
- Hit **2ND** [STAT PLOT]
- Choose 1: Plot1 by either hitting **ENTRY** or **ENTER**.
- Turn **On** the plot by pressing **ENTER**.
  - Next to **Type**, you should have selected **scatterplot**
  - For **Xlist**, you should have L₁
  - For **Ylist**, you should have L₂
  - For **Mark**, you may choose any of the three options to represent the points on your scatterplot
- Hit **ZOOM** and choose 9: ZoomStat by scrolling down to 9 and hitting **ENTER** or by simply hitting **9** to view the scatterplot.

*If the pattern of the data is appropriate for linear regression, continue with the following step.*

**STEP 3: Getting the regression equation (and storing it into the equation editor)**
- Hit **STAT** then **4** to CALC
- Choose 4: LinReg(ax+b) (Either scroll down to 4 and then hit **ENTER**, or simply hit **4**)
- Hit **VARS** then **2** to Y-VARS
- Choose 1: **Function** by hitting **ENTER**
- Choose 1: **Y₁** by hitting **ENTER**
- Hit **ENTER**

The coefficients of your linear regression equation (a and b) will be displayed on your homescreen. The linear regression equation will be stored in the equation editor in Y₁.

*Note: The directions in Step 3 refer to linear regression. If a different type of regression is more appropriate, replace 4:LinReg(ax+b) with the more appropriate regression type found in the **STAT** **1** CALC menu.*
Correlation and Best-Fitting Lines

Tell whether x and y have a positive correlation, a negative correlation, or relatively no correlation.

Example

WORLD POPULATION. The following table gives the United Nations estimates of the world population (in billions) every five years from 1980 - 2005.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>4.451</td>
</tr>
<tr>
<td>1985</td>
<td>4.655</td>
</tr>
<tr>
<td>1990</td>
<td>5.295</td>
</tr>
<tr>
<td>1995</td>
<td>5.719</td>
</tr>
<tr>
<td>2000</td>
<td>6.124</td>
</tr>
<tr>
<td>2005</td>
<td>6.515</td>
</tr>
<tr>
<td>2010</td>
<td>?</td>
</tr>
</tbody>
</table>

Source: UN 2006 Revisions Population database

Step 1) Use your calculator to make a scatter plot. Then describe the correlation shown by the scatter plot.

Step 2) Use your calculator to approximate the best-fitting line for the data. \( y = ax + b \)

Step 3) What is the correlation coefficient and what does it mean for your data?

Step 4) Use your calculator to graph the best-fitting line. Use your calculator to predict the population in 2013.
**ACTIVITY: HEIGHT AND SHOE SIZE**

1. Record your shoe size and height in inches on a slip of paper. Because sizing is different for men and women, female students will need to record the corresponding men’s size if they know it or to subtract 1.5 from their women’s size. Record the results from the class in the tables below.

<table>
<thead>
<tr>
<th>Height</th>
<th>Shoe size</th>
<th>Predicted Shoe Size</th>
<th>Amount of Miss</th>
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<td></td>
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2. Now for each person, plot the pair (height, shoe size) as a point on the grid.

3. Do most of the points seem to come close to falling on a line? One way to tell is to draw as narrow an oval as possible around the points. The narrower the oval, the closer the points are to lining up. The more circular the oval, the less so.

4. Draw a line through the middle of your scatter plot. The line should also run through the middle of your oval.
5. Your line is a *geometric model* for the relationship between height and shoe size.

   You can use your model to estimate your shoe size. Find your height on the horizontal scale and move upward until you come to the line. Then move to the left until you come to the vertical scale. Read your predicted shoe size.

   What shoe size was predicted? ________________

   How does the predicted shoe size compare with your actual shoe size? ____________

6. Choose two points on your line whose coordinates are integers. They do not have to be points of your original data but should be fairly far apart. Record the coordinates of the points below:

   (____,____)  (____,____)

7. Use the two points to find the slope of the line.__________

8. Use the slope and one of your two points to find the y-intercept \((0, b)\) of the line. Write the y-intercept and the equation of the line below:

   y-intercept:_______

   equation:_________________________________

9. Your equation is an *algebraic model* for the relationship between height and shoe size.

   Use this model to calculate your predicted shoe size. Do this by substituting your height for \(x\) in the equation and rounding the shoe size to the nearest 0.5

   What shoe size was predicted? ____________

   How close is the predicted size to your real shoe size? ____________

10. Use your equation to calculate the predicted shoe size for each member of your class. Be sure to round to the nearest 0.5. Record the results in the third column of the table. In the fourth column, record the amount the predicted size missed the real size for each student.

11. Compare your misses with those of another student. Whose line did the best job of predicting shoe sizes for members of your class? Why?

   ______________________________________________________________________

   ______________________________________________________________________

12. Use your graphing calculator to create a scatterplot similar to the plot in #2. Enter Data in stat menu and adjust window settings.
13. Use your graphing calculators linear regression feature to create and graph an accurate linear model of the data.

What is your linear regression equation? ___________________________

How does it compare to your equation in #8? Is it more accurate or less accurate? Why or why not?
____________________________________________________________________

What does your regression equation predict your shoe size to be? Close? __________ Yes  No

14. a. According to the Guinness Book of World Records, the tallest person who ever lived was Robert Wadlow, who was 8 feet, 11 inches tall.

   Predict his shoe size using your equations.
   #8 equation__________   Calculator__________

b. The tallest living woman, Sandy Allen, is 7 feet, 7 inches.

   Predict her shoe size using your equations
   #8 equation__________   Calculator__________

15. What shoe size do your equations predict for a person who is 50 inches tall?______________

   Do you think this prediction is reasonable? Why or why not?
____________________________________________________________________

16. Based on Shaquille O'Neal's footprint, could you predict his height using your equation? Yes No

   Is this accurate? Yes  No   Why or why not?
____________________________________________________________________

17. Do you think it would be reasonable to sell shoes by a person's height? Yes  No   Why or why not?
____________________________________________________________________

18. What generalizations can you make from this activity in regards to modeling, or more specifically, mathematical modeling?
____________________________________________________________________

____________________________________________________________________

____________________________________________________________________
Activity: CBL Walking Graphs

1. In your group, come up with a strategy to create each of the graphs below. The CBL will record a person’s distance from the motion detector as a function of time.

2. Each group will turn in...
   a) A sentence for each graph describing how a person(s) must walk in order to create each graph.
   b) A different scenario (2 quantities) other than distance vs. time that each graph could be used to describe. Define what 2 quantities have this relationship and justify your answers.

For example, for a straight line with positive slope and y-intercept of 0: A person must start right in front of the motion detector and walk with a constant speed away from the detector. This graph may also be used to represent the amount of money someone earns (on the dependent axis) as a function of the number of hours she worked (independent axis). The slope would represent the person’s wage.