Isosceles and Equilateral Triangles

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<tr>
<td>If two sides of a triangle are congruent, then the angles opposite the sides are congruent.</td>
<td>If ( RT \cong RS ), then ( \angle T \cong \angle S ).</td>
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<tr>
<td><strong>Converse of Isosceles Triangle Theorem</strong></td>
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<td>If two angles of a triangle are congruent, then the sides opposite those angles are congruent.</td>
<td>If ( \angle N \cong \angle M ), then ( LN \cong LM ).</td>
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</table>

You can use these theorems to find angle measures in isosceles triangles.

**Find \( m\angle E \) in \( \triangle DEF \).**

\[
m\angle D = m\angle E \quad \text{Isosc. } \triangle \text{ Thm.}
\]

\[
5x^\circ = (3x + 14)^\circ \quad \text{Substitute the given values.}
\]

\[
2x = 14 \quad \text{Subtract } 3x \text{ from both sides.}
\]

\[
x = 7 \quad \text{Divide both sides by } 2.
\]

Thus \( m\angle E = 3(7) + 14 = 35^\circ \).

**Find each angle measure.**

1. \( m\angle C = \) __________________

2. \( m\angle Q = \) __________________

3. \( m\angle H = \) __________________

4. \( m\angle M = \) __________________
**Reteach**

**Isosceles and Equilateral Triangles continued**

**Equilateral Triangle Corollary**
If a triangle is equilateral, then it is equiangular.

(equilateral $\triangle \rightarrow$ equiangular $\triangle$)

**Equiangular Triangle Corollary**
If a triangle is equiangular, then it is equilateral.

(equiangular $\triangle \rightarrow$ equilateral $\triangle$)

If $\angle A \equiv \angle B \equiv \angle C$, then $\overline{AB} \equiv \overline{BC} \equiv \overline{CA}$.

You can use these theorems to find values in equilateral triangles.

**Find $x$ in $\triangle STV$.**

$\triangle STV$ is equiangular. 

$\measuredangle V = 60^\circ$

The measure of each $\angle$ of an equiangular $\triangle$ is $60^\circ$.

$7x + 4 = 60$

Subtract 4 from both sides.

$7x = 56$

Divide both sides by 7.

$x = 8$

**Find each value.**

5. $n =$  

6. $x =$  

7. $VT =$  

8. $MN =$
9. 693 ft  
10. 50°
11. 6.3  
12. 60°
13. $4\frac{1}{2}$ yd  
14. 65°
15. 8

Practice B

1. Possible answer: It is given that \( \overline{HI} \) is congruent to \( \overline{HJ} \), so \( \angle I \) must be congruent to \( \angle J \) by the Isosceles Triangle Theorem. \( \angle IKH \) and \( \angle JKH \) are both right angles by the definition of perpendicular lines, and all right angles are congruent. Thus by AAS, \( \triangle HKI \) is congruent to \( \triangle HKJ \). \( \overline{IK} \) is congruent to \( \overline{KJ} \) by CPCTC, so \( \overline{HK} \) bisects \( \overline{IJ} \) by the definition of segment bisector.

2. 58.1 ft  
3. 45°
4. \( \sqrt{2} \)
5. 36 or 9
6. 76°  
7. \( \frac{4}{3} \)
8. 10  
9. 30°
10. 89

Practice C

1. Possible answer: \( \triangle ABC \) is an isosceles triangle with vertices \( A(0, b) \), \( B(a, 0) \), and \( C(-a, 0) \). \( D \) is the midpoint of \( \overline{BC} \), so \( D \) has coordinates \((0, 0)\). The slope of \( \overline{AD} \) is \( \frac{b - 0}{0 - (-a)} = \frac{b}{a} \), so the slope is undefined. A line with an undefined slope is a vertical line. The slope of \( \overline{BC} \) is \( \frac{0 - 0}{a - (-a)} = 0 \). A line with a zero slope is a horizontal line. Because \( \overline{AD} \) is vertical and \( \overline{BC} \) is horizontal, \( \overline{AD} \perp \overline{BC} \).

2. \( x = 36; y = 72; z = 36 \)
3. \( 2\ell \)

4. Possible answer: The coordinates of \( Y \) are \((a, \sqrt{3}a)\). The Midpoint Formula shows that the midpoints of the sides of \( \triangle XYZ \) are \( A\left(\frac{1}{2}a, \frac{\sqrt{3}}{2}a\right), B\left(\frac{3}{2}a, \frac{\sqrt{3}}{2}a\right) \), and \( C(a, 0) \). The Distance Formula gives these distances: \( AX = AY = AC = AB = BC = XC = BY = BZ = CZ = a \). Thus by SSS, \( \triangle ABC \cong \triangle XAC \cong \triangle YAB \cong \triangle CBZ \), and the triangles are equilateral.

Reteach

1. 51°  
2. 47°
3. 72°  
4. 60°
5. 12  
6. 33
7. 18  
8. 13

Challenge

1. \( x = 10; \) By Isosc. \( \triangle \) Thm. and \( \triangle \) Sum Thm., \( m\angle TSV = 54° \). \( \triangle QSV \equiv \triangle TSV \) by SSS, so \( m\angle VSQ = 54° \) by CPCTC. \( m\angle QSR = 72° \) by Lin. Pair Thm. and by \( \angle \) Add. Post. By Isosc. \( \triangle \) Thm. and \( \triangle \) Sum Thm., \( m\angle SQR = 36° \). Solve \((3x + 6)° = 36°\). \( x = 10 \)

2. 42.2; \( \triangle FJH \) is an equilat. \( \triangle \), so \( HJ = JF = FH = 15 \) and \( x = 3 \). So \( GJ = 12.2 \). \( \overline{FG} \equiv \overline{FJ} \) since the sides opp. the \( \angle \) are \( \angle \equiv \). So \( FG = FJ = 15 \). \( \angle EFH \equiv \angle GFJ \) by AAS, so the corr. sides are \( \equiv \) by CPCTC. \( P = 12.2 + 15 + 15 = 42.2 \)

3. 108°; \( \triangle FJH \) is equiangular, so \( m\angle JFH = m\angle FJH = 60° \). By Lin. Pair Thm. and \( \angle \) Add. Post., \( m\angle FJG = 66° \). By Isosc. \( \triangle \) Thm. and \( \triangle \) Sum Thm., \( m\angle GFJ = 48° \). \( m\angle GFH = 48° + 60° = 108° \)

Problem Solving

1. 14 in.  
2. 40°
3. 11 ft; \( m\angle GJH = 72° - 36° = 36° \). \( m\angle GHJ = 36° \) by Alt. Int. \( \angle \) Thm. By Converse of Isosceles Triangle Theorem, \( GJ = GH = 11 \) ft.